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P081: FINITE ELEMENT STABILIZATION METHODS AND SOLVERS FOR HEAT EXCHANGER APPLICATIONS: A REVIEW

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Abstract

This review focuses on the applications of finite element method (FEM) for heat exchanger analyses. Solutions to convection-dominated heat transfer problems usingtheGalerkinFEM approximation are always characterised with errors caused by numerical instabilities. Efforts to enhance the stability and exactness of results had led to development of a number of stabilization techniques.Also, there have been algorithms formulated to effectively solve the sparse symmetric and non-symmetric matrix systems resulting from FEM discretised equations of thermal flow problems. The development of stabilization techniques and solvers has made the FEM approach a more formidable computational fluid dynamics (CFD) tool. However, there have been limited uses of finite element CFD codes to heat exchanger applications.
Keywords: Finite element method, Heat exchanger, Stabilization techniques, Solvers

1.0 Introduction

Heat exchangers are widely used in industries to cause transfer of heat between two or more fluids. Common applications of heat exchangers are found in power plants, space heating, automotive cooling systems, heat recovery units, food and process industries and so on (Bhutta et al., 2011; Nasiruddin and Siddiqui, 2006; Parikshit et al., 2015). Heat exchanger (HE) types found in these industries are not limited to spiral, lamella, plate-fin, tube-fin, shell-and-tube, double pipe, spiral, rotary and fixed matrix heat exchangers (Bhutta et al., 2011).

According to Borah et al. (2013), the heat transfer process between a hot and a cold fluid in any heat exchanger is achieved through mechanisms of conduction and convection. However, the numerical predictions of the thermal dynamic phenomena, which are described by partial differential equations (PDEs), within heat exchangers, are complex to resolve (Micheletti et al., 2005).

The advent of digital computer and easy-of-use powerful software packages, the subject of computational fluid dynamics (CFD), which employs the computational or numerical techniques to solve PDEs, has increasingly become interesting and as it offers solutions to problems difficult to handle by analytical and experimental methods in more convenient ways (Cengel and Ghajar, 2015; Papanastasiou et al., 2000). This interest has been extended to heat exchanger analysis (Nithiarasu, 2005). Full descriptions and details of flow field variables are usually available at any region of any computational domain in CFD as repeated and parametric runs of numerical models are usually accomplished with reduced efforts (ALENTEC, 2014).

As stated by Kotwal and Patel (2013), the alternative, cost effective and speedy solutions to heat exchanger designs and optimizations are offered by CFD. Also, Stevanović et al. (2001) added that owing to the energy efficient and low cost design provided by this method, researchers have successfully employed it in predicting complex flow and heat processes over tube bundles in heat exchanger.

In this paper, a critical review of finite element CFD technique and its solvers for analyses of heat exchangers will be discussed.

2.0 Numerical Solutions to Mathematical Models

There are several discretisation methods employ in finding solutions to problems established on partial differential equation based mathematical models (Nithiarasu, 2005; Peiro and Sherwin, 2005). This is necessitated because of the closure problem caused by more number of unknowns to available number of PDE equations, and a large number of approximate discretised algebraic equations derived with any of these methods are solved for the unknowns (Sayma, 2009).

The oldest and simplest in implementation among these numerical methods is the finite difference method (FDM). FDM makes use of the truncated Taylor's expansion on structured rectangular grids to approximate solutions to problems (Kuzmin, 2010; Peiro and Sherwin, 2005; Sayma, 2009). But the use of this method has been limited by difficulty it normally encounters with irregular geometries (Lewis et al., 2004; Peiro and Sherwin, 2005).

The finite volume method is also a numerical method which has found its place in commercial computational fluid dynamics (CFD) codes. This method extends the use of FDM for both structured and unstructured meshes to solve complex flow problems using integral formulation (Kuzmin, 2010; Lewis et al., 2004). FVM is very versatile in discretising hyperbolic equations but its accuracy reduces whenever approximating diffusive fluxes, also the high-order estimations of FVM are very hard to formulate using the theories that established its first and second order schemes (Kuzmin, 2010).

Comparing with FDM and FVM, a relatively new numerical method in CFD analysis is the finite element method (FEM), although it has been extensively used for structural analysis problems (Gikadi, 2013; Kuzmin, 2010; Lewis et al., 2004; Sayma, 2009). FEM as with FVM is capable of approximating problems with irregular geometries using integral formulations on structured and unstructured meshes (Sayma, 2009). Although, FEM has been described to offers the best solutions to elliptic and parabolic problems in structural analysis, it requires suitable technique to capture the hyperbolic nature of convection-dominated flows (Gikadi, 2013; Kuzmin, 2010).

According to (Lewis et al., 2004), several approaches to finite element analysis have been developed such as the Ritz (heat balance integral), Rayleigh-Ritz (Variational) and the weighted residual methods. Also, the commonly used techniques of the weighted residual method are the point collocation method, sub-domain collocation method, least-square method and the most popular Galerkin method (Sayma, 2009; Zienkiewicz and Taylor, 2000).

There exits some variants of the FEM, among these is the Spectral Element Method (SEM). SEM combines the accuracy of pseudo-spectral method with the flexibility of Galerkin FEM for complex geometries, and has been found to be an effective tool for solving frequency domain wave propagation problems (Komatitsch et al., 2005; Schuberth, 2003). One other variant of FEM is the boundary element method (BEM). BEM has advantage where there are very large or infinite elements; therefore, solutions found from the exterior domains are integrated over to the interior to determine the solutions of the unknowns. This method has been described to be more applicable to nonlinear problems and smooth boundaries for accuracy (Aliabadi and Wen, 2011; Antes, 2010).

Another numerical method in the field of numerical science is the vortex element method (VEM), this method has recorded good success in analysing complex fluid and heat transfer problems using grid-free attribute with simple mathematical operations at lower computational memory and time (Dare and Petinrin, 2010; Petinrin et al., 2010). But this method has not been widely developed into CFD code.

3.0 Numerical Stabilization of Convection-Dominated Flows

Solutions offer by Galerkin approximation to convection-dominated transport problems are always characterised with errors (Zienkiewicz and Taylor, 2000), and this is as a result of the spurious oscillations caused by numerical instabilities (Fries & Matthies, 2004; Knobloch, 2007; Lewis et al., 2004; Matthies et al., 2015). Although, the same problem is also found with the FDM and FVM but using upwind schemes in reducing the oscillations (Lewis et al., 2004). Peclet and Reynolds numbers are the two important dimensionless parameters that are used to check the onset of instability with Galerkin FEA approach, and the higher any of these numbers the more instability in the system (Fries and Matthies, 2004). For linear elements, oscillation begins when the element Peclet number or Reynolds number is greater than one, and given one-dimensional PDE-based equation as

$$\frac{\partial \phi}{\partial t}+u\frac{\partial \phi}{\partial x}-\frac{\partial}{\partial x}\left(c\frac{\partial \phi}{\partial x}\right)=0 \tag{1}$$

The element Peclet number, *Pe* is determined by (Fries and Matthies, 2004; Gikadi, 2013; Nigro et al., 2015)

$$Pe=\frac{uh}{2c} \tag{2}$$

From equations (1) and (2), ϕ is the flow variable, u is the convective velocity vector, c is the diffusion coefficient and h is the element size in the flow direction, and for one-dimensional problem it is the length of the local element.

From on-going, the instability can be reduced by decreasing the element size, but the practicability is very rare as it requires a very high density mesh generation (COMSOL, 2013; Sun et al., 2009).

However, there are a number of stabilization techniques that have been developed for enhancing the stability and exactness of results analysis with high Peclet number (John and Knobloch, 2016; Lewis et al., 2004; Matthies et al, 2015; Sun et al., 2009).

One of the earliest approaches to eliminate the instabilities characterised by the standard Galerkin method in the 1970s is the introduction of the artificial diffusion scheme. With this scheme an artificial viscosity term was introduced in the equation to reduce the Peclet number and eliminate the negative diffusion caused by numerical instability (Fries and Matthies, 2004; Gikadi, 2013). (Gikadi, 2013) further stated that having a reasonable selection of the artificial diffusion, this approach is very accurate for one-dimensional problem but predictions are always inaccurate as space dimension increases because of excessive and isotropic additional diffusion caused by artificial diffusion term. Also, inaccuracies set in for inability of this approach to specifically take care of the source term whenever present (Fries and Matthies, 2004; Gikadi, 2013).

The Petrov_Galerkin approach modifies the weight functions, thus making it different from the shape functions (Fries and Matthies, 2004; Onate and Manzan, 2000). Therefore, for a PDE of general form

$$Lu=f \tag{3}$$

and weak form

$$\int_{\Omega} w^{*}\left(L\tilde{u}-f\right)d\Omega=0 \tag{4}$$

This weight function can then be determined as

$$w_i^{*}=w_i+\tau w_i' \tag{5}$$

Here the w_i is the original shape function, τ is the stabilization parameter and w'_i is the new test function for node i and L is advection-diffusion-reaction or any differential operator (Chaple, 2006; Onate and Manzan, 2000; Zienkiewicz and Taylor, 2000). However, effect of adding crosswind diffusion, another technique that introduces artificial diffusion in streamline direction for capturing of undershoots and overshoots, is always felt on the output results while solving time-dependent and multi-dimensional problems (Fries and Matthies, 2004).

To overcome the inherent problems associated with the isotropic artificial diffusion, the streamline-upwind Petrov-Galerkin (SUPG) was developed by Brooks and Hughes in 1979 (Fig. 1) (Franca et al. 2003; Fries and Matthies, 2004; Gikadi, 2013; Knobloch, 2007; Lewis et al., 2004). As further explained by (Fries and Matthies, 2004; Gikadi, 2013; Kuzmin, 2010), SUPG introduces artificial diffusion tensor instead of a scalar only in the streamline direction thereby imposing consistency as against the inconsistency of the isotropic artificial diffusion schemes and this makes the artificial diffusion terminology not suitable for this approach. Here the weight function is defined as

$$w_i^* = w_i + \tau L_{adv} w_i \qquad (6)$$

From equation q, L_{adv} is the advective part of the operator (Chaple, 2006; Fries and Matthies, 2004).

According to (Gikadi, 2013), Brooks and Hughes in 1982 modified the technique by applying this operator on all the equation terms to insure consistency.

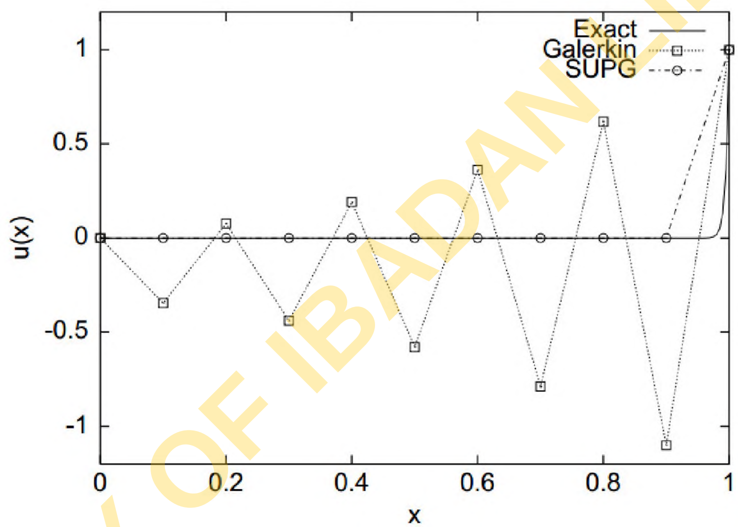


Fig. 1: A result from spurious oscillation with Galerkin method (Franca et al., 2003)

Galerkin Least-Square (GLS) scheme is another technique which was stemmed from mathematical relations and shares the same characteristics and equality with the SUPG for convection-diffusion problems with linear elements (Fries and Matthies, 2004; Gikadi, 2013; Onate and Manzan, 2000). But this scheme as against the SUPG includes the diffusive and reactive terms in its stabilization operator (Gikadi, 2013). The GLS adds the least square form of residuals to the Galerkin method to boost its stability without compromising the consistence and accuracy of results (Fries and Matthies, 2004). Also for GLS, the weight function can be given as (Chaple, 2006; Fries and Matthies, 2004).

$$w_i^* = w_i + \tau L w_i \qquad (7)$$

Pressure Stabilizing Petrov-Galerkin (PSPG) is also a technique used to dampen the instability caused by the Lagrange multiplier that is the pressure through the perturbation of the test functions (Fries and Matthies, 2004; Chaple, 2006). This approach and others like the SUPG and GLS disregard the popular Babuska-Brezzi condition which has to do with the substitution of the pressure term with velocity interpolation (Franca et al, 2003; Fries and Matthies, 2004; Lewis et al, 2004).

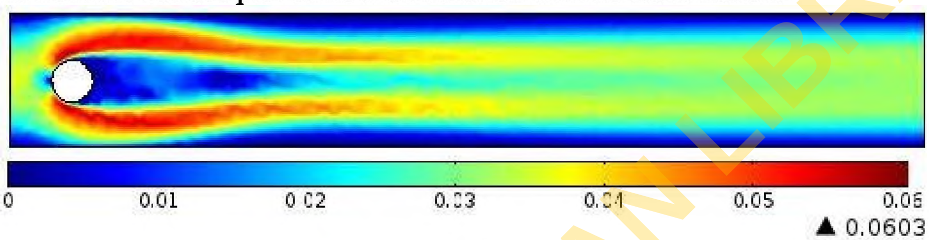
Onate and Manzan (2000) reported that the subgrid scale (SGS) method is an all-purpose scheme to generate different kinds of stabilisation techniques, similar to GLS but its operator has a difference sign in the diffusive term. The weight function here is (Chaple, 2006).

$$w_i^* = w_i + \left(-L^*\right)\tau w_i' \tag{8}$$

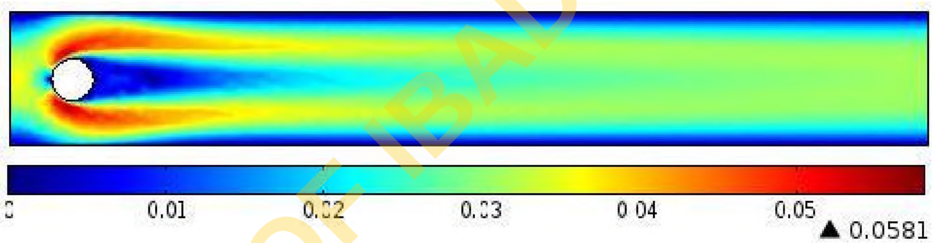
where L^* is the adjoint operator

Other schemes that have been reported in literatures that are have been successfully for time-dependent problems are the Taylor–Galerkin (TG) and Characteristic Galerkin (CG). The CG, which is also known to as the Characteristic Based Split (CBS) technique, follows Babuska-Brezzi condition and has a wider applicability (Lewis et al, 2004; Zienkiewicz and Taylor, 2000).

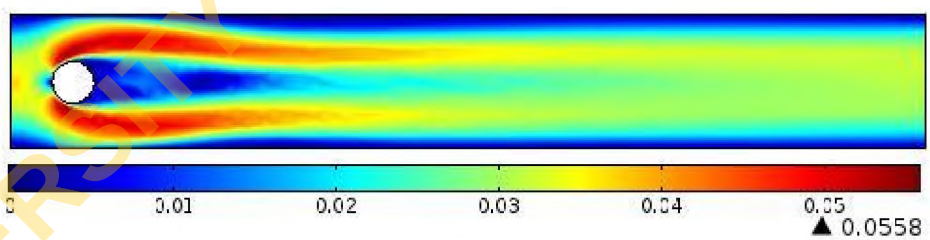
Different combinations of these stabilization techniques have been used to obtain more accurate solutions, most especially the SUPG/PSPG (Fries and Matthies, 2004; Tezduyar and Sathe, 2003). A typical illustration is shown in Fig. 2 for an air flow around a cylinder located at 0.15 m from entry of a channel (0.32 x 2.20 m). It is clearly shown that spurious oscillations are more evident from the surface plot of flow distribution with no stabilization.



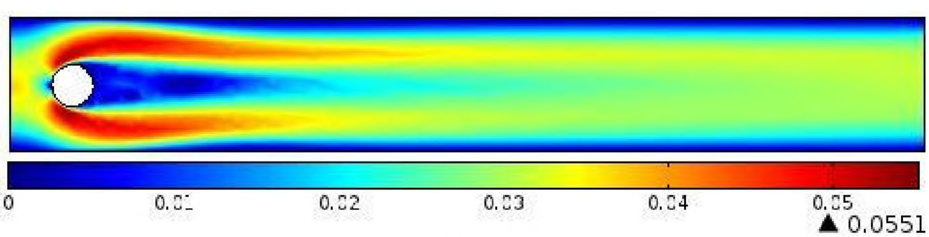
(a)



(b)



(c)



(d)

Fig. 2: Flow of velocity distribution around a cylinder (a) no stabilization, (b) with crosswind diffusion, (c) with streamline (SUPG/GLS) diffusion and (d) with both streamline (SUPG/GLS) and crosswind diffusions

4.0 Finite Element Discretised Equation Solvers

In the last two decades, different algorithms have been formulated in order to effectively solve both sparse symmetric and non-symmetric matrix systems (Pozza, 2013). Ways of solving finite element discretised equations which are always characterised by these kinds of matrix systems for convection-dominated problems are divided into two, namely: direct and iterative methods, and the choice of any solver depends on the desired accuracy, time and memory requirements (Peladeau-Pigeon, 2012).

The direct solvers make use of makes use of the Cholesky factorization, LU factorization or the Gaussian Elimination to provide exact solutions to system of linear equations in a number of finite sequences while trading off with the round-off errors. These solvers are very robust and deterministic and sometimes provide solutions to complicated problems at very high computer memory costs (Gikadi, 2013; Peladeau-Pigeon, 2012; Pozza, 2013).

As reviewed by Ferfecki (2013) and Pozza (2013), one of the common direct solvers is the Multifrontal Massively Parallel Sparse (MUMPS) direct solver, which is fast and highly efficient for both symmetric and non-symmetric system of matrices, shared memory multi-core ability and cluster capable, and another solver which shares almost the same features with MUMPS is the Parallel Sparse Direct Solver (PARDISO). Ferfecki (2013) also commented on yet another direct solver which is Sparse Object Oriented Linear Equations Solver (SPOOLES) that it requires less computer memory but slower in computing than MUMPS. Some other direct solvers are the Frontal and Multifrontal solvers but are less efficient than MUMPS (Pozza, 2013).

The iterative solvers employ an initial guess to find good approximations to exact solutions by improvement on repetitive and sequential estimations. Although, the iterative solvers are less efficient as the convergence levels may be far from exact solutions and sometimes take more computational time but they require less computer memory and become more unavoidable to use as the spatial direction increases to three (Gikadi, 2013; Peladeau-Pigeon, 2012; Pozza, 2013). Also, Pozza (2013) added that the dependability and improvement on the convergence of any iterative solver can be achieved with the use of a good preconditioner, which is employed as transformation matrix to modify the arrangement of the coefficient matrix and smoothen the solution of the solver.

Among the most common iterative schemes are the Jacobi, Gauss-Seidel, successive over-relaxation (SOR) and the symmetric successive over-relaxation (SSOR) solvers, very simple in implementation and robust but are less effective for practical problems as iteration time increases out of proportion for more number of degrees of freedom (Gikadi, 2013; Pozza, 2013).

According to Ferfecki (2013), Gikadi (2013), Kirkegaard and Auken (2015) and Pozza (2013), the conjugate gradient (CG) solver method is one of the more efficient and cluster capable solvers, but only effective for hermitian and positive definite matrices. Extending the capability of CG to handle the non-hermitian and indefinite matrices, the bi-conjugate gradient (BiCG) and bi-conjugate gradient stabilized (BiCGStab) were developed. But COMSOL (2013) and Gikadi (2013) stated that the convergence behaviour of both solvers may be erratic and stop before finding a good approximate solution, this is however still fair for the BiCGStab.

Also, another set of more robust but with more memory requirements than the BiCGStab are the generalized minimal residual (GMRES) and the flexible generalized minimal residual (FGMRES), which as well handles more preconditioners and requires more memory than the GMRES (COMSOL, 2013; Ferfecki, 2013; Gikadi, 2013).

While these iterative solvers have been categorised as Krylov subspace methods, the preconditioners normally apply to aid their performance and accelerate convergence still include the usualJacobi, Gauss-Seidel, symmetric successive over-relaxation (SOR) and symmetric successive over-relaxation (SSOR) (Pozza, 2013). Other preconditioners that have been successfully used are not limited to the single-level symmetric Gauss-Seidel (SGS) relaxation, algebraic multigrid (AMG), geometric multigrid (GMG) and incomplete LU (Ferfecki, 2013; Gikadi, 2013; Pozza, 2013).

5.0 Finite Element Based Software Codes and Their Heat Exchanger Applications

Several finite element codes have been built for modelling of flow and heat transfer problems, which are also capable of modelling the thermal and hydraulic characteristics of heat exchangers. Some of the codes are FIDAP, POLYFLOW, Elmer, Modelica, Abaqus, ADINA, COMSOL Multiphysics and CalculiX (Bhutta et al., 2011; Borah et al., 2013). Although, the codes have been used extensively to conduct flow and heat transfer analyses, literature are scarce on their heat exchanger applications. However, a number of researchers have used some of the finite element CFD codes for heat exchanger analyses as shown in Table 1. In a compressive review given by Bhutta et al. (2011) on CFD application to heat exchangers design, almost all the works with CFD codes listed were built on FVM, and they were FLUENT and CFX. This does not imply that finite volume solvers are better at approximating heat exchanger problems than solvers built on FEM. A test case conducted by Micheletti et al. (2005), indicated a more accurate prediction of FEM over FVM for the same number of elements while the FEM with grid adaptation gave the best result for specific enthalpy of heat exchanger outlet as shown in Fig. 3. Thus, precision level of any CFD solution does not depend only on the numerical methods of choice, but also on the selected stabilization technique, solver, mesh grid type and adaptation, and so on.

Table 1: Heat exchanger analyses with finite element CFD codes

Authors	HE Type	CFD Code	Stabilization Technique and Solver
Casella and Schiavo (2003)	Generic	Modelica	GLS
Charvátová et al. (2013)	Counterflow tube	COMSOL Multiphysics	-
Desgrosseilliers and Groulx (2014)	Double pipe	COMSOL Multiphysics	-
Guillaume (2011)	Coaxial borehole	COMSOL Multiphysics	-
Fernandes et al. (2005)	Chevron plate	POLYFLOW	-
Fernandes et al. (2007)	Chevron plate	POLYFLOW	An iterative method
Hameed and Essa (2015)	Finned tube	COMSOL Multiphysics	-
Jia et al. (2014)	Counter flow parallel-plate	COMSOL Multiphysics	-
Jia et al. (2014)	Plate Fin	COMSOL Multiphysics	-
Lele et al. (2014)	Plate-fin and Helical coil	COMSOL Multiphysics	-
Micheletti et al. (2005)	Generic	Modelica	GLS
Petinrin and Dare (2015)	Shell-and-Tube	COMSOL Multiphysics	Segregated solvers: Two GMRES and

			MUMPS
Pryor (2014)	Crossflow	COMSOL Multiphysics	-
Reddy et al. (2012)	Shell-and-Tube	COMSOL Multiphysics	Segregated solvers
Saffarian et al. (2014)	Shell-and-Tube	COMSOL Multiphysics	-
Yakah (2012)	Plate-fin and Printed circuit	COMSOL Multiphysics	-

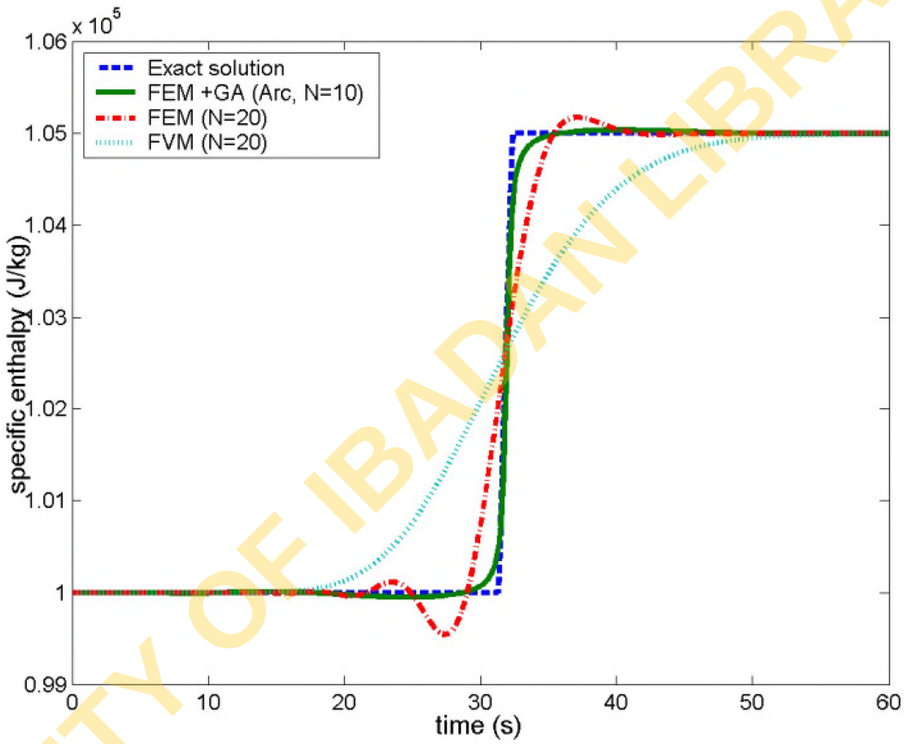


Fig. 3: Specific enthalpy at HE outlet (Micheletti et al., 2005)

Conclusion

The full description of the mechanism of heat transfer processes in heat exchangers are defined by partial differential equations. Computational fluid dynamics offers the best alternative, cost effective and speedy solutions in predicting complex flow and heat processes to heat exchanger designs and optimizations owing to the energy efficient and low cost design. A number of stabilization techniques and efficient solvers have been developed to enhance the accuracy of solutions offer by finite element method, one of the CFD tool, for effective prediction convection-dominated flows. However, from available number of previous studies on numerical simulation of heat exchangers with CFD codes, applications of finite element based software are still limited in relation to the finite volume based codes. This may be due to its late arrival to CFD modelling applications.

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