EVALUATION OF SMALL SAMPLE ESTIMATORS OF OUTLIERS INFESTED SIMULTANEOUS EQUATIONS MODEL: A MONTE CARLO APPROACH

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Abstract

In practice, data collected in a broad range of applications frequently contain one or more *atypical observations* called *outlier.* A single outlier can have a large distorting influence on a classical statistical method that is optimal under the assumption of normality or linearity. Many estimation procedures proposed by researchers to handle simultaneous equation models are based on the assumptions that give little consideration to atypical data, thus the need to investigate the distorting effects of outliers in simultaneous equations estimation methods.

In this study, we compare the performance of five estimators (OLS, 2SLS, 3SLS, GMM and W2SLS) of simultaneous equations model parameters at small sample sizes (n) 15, 20 and 25; first order autocorrelation levels (ρ) 0.3, 0.6 and 0.9 of the error terms, when the series are perturbed (polluted) at zero, one and two times. The estimators are adjudged using the minimum criteria of Bias, Variance and RMSE criteria on the 135 scenarios, each replicated 10,000 times.

Identical results were obtained for the 2SLS and W2SLS methods since there are no restrictions on the parameters. The system methods clearly performed better than the single equation counterparts. Generally, the estimates obtained for the just identified equation are better than those of the over identified counterpart. Surprisingly, the ranking of the various techniques on the basis of their small sample properties does not reveal any distinguishable feature according to whether there is outlier(s) in the data or not and at the different level of correlation. On the BIAS criterion, the best method is OLS in the just identified equation, followed by 3SLS in most cases especially where the pollution level is zero for all the three autocorrelation levels considered. The GMM and 2WSLS are struggled for the third and last positions. However, in the over identified case, 3SLS is leading closely followed by GMM in most cases (when rho is 0.9 for all sample sizes considered) and OLS in few other cases (especially at rho = 0.3 and 0.6 and for N = 20 and 25 with single/double pollution levels), it is expected that we would be able to identify or suggest the best method to use when we have the scenario depicted above.

Keywords: Outlier, Small sample, Simultaneous Equations, Autoregressive Error Terms

Introduction

In practice, data collected in a broad range of applications frequently contain one or more *atypical observations* called *outliers*; that is, observations that are well separated from the majority or "bulk" of the data, or in some way deviate from the general pattern of the data. A single outlier can have a large distorting influence on a classical statistical method that is optimal under the assumption of normality or linearity. The presence of outlier in a data set can lead to inflated error rates and substantial distortions of parameter and statistic estimates when using parametric or nonparametric test (Zimmerman 1998). As a matter of fact, the effects of outliers will pervade through all the equations and the estimated structural parameters in them. These effects are so intricately pervasive that it is very difficult to assess the influence of outliers on the estimated structural parameters (Mishra, 2008). Osborne et al (2001) confirmed empirically that researchers rarely report checking for outliers of any sort, by reporting that authors reported testing assumptions of the statistical procedure(s) used in their studies, including checking for the presence of outliers, only 8% of the time.

Many estimation procedures have been proposed by researchers to handle simultaneous equation models. These procedures are based on the assumptions that stochastic terms be normally distributed and existence of zero correlation between pairs of random deviates. These assumptions give little consideration to atypical data, thus there is the need to investigate the distorting effects of outliers on each of the methods and determine the best estimation procedure under the influence of outliers and when the errors are not well behaved. Any relationship of econometric theory will almost certainly belong to a system of simultaneous equations whose parameters may be estimated by various simultaneous equation estimation techniques. The problem frequently faced is the choice of the best estimation technique.

To assess the quality and appropriateness of estimators, we are always interested in their statistical properties. For most estimators, these can only be derived in a "large sample" context, (*asymptotic properties*). One estimation procedure may, for example, be selected over another because it is known to provide consistent and asymptotically efficient parameter estimates under certain stochastic environments. Such a heavy reliance on asymptotic theory can and does lead to serious problems of bias (in estimation) and low levels of inferential accuracy when sample sizes are small and asymptotic formulae poorly represent sampling behaviour. This has been acknowledged in mathematical statistics since the seminar work of R. A. Fisher (1925), who recognised very early the limitations of asymptotic machinery, when he wrote; *"Little experience is sufficient to show that the traditional machinery of statistical processes is wholly unsuited to the needs of practical research. Not only does it take cannon to shoot a sparrow, but it misses the sparrow! The elaborate mechanism built on the theory of infinitely large samples is not accurate enough for simple laboratory data. Only by systematically tackling small sample problems on their merits does it seem possible to apply accurate tests to practical data" (Olaomi and Shangodoyin, 2010)*

2.0 The Model

Simultaneous equations models have the form

$$\mathrm{B} y_i = \Gamma x_i + e_i, \qquad i = 1, \ldots, n \tag{1}$$

where $x_i \in R^p$ are the vectors of *exogenous variables*, $y_i \in R^q$ are the *endogenous variables*, and the disturbances $e_i \in R^q$ are i.i.d random vectors with mean 0 and covariance matrix Σ. The matrices $\mathrm{B} \in R^{2\times 2}$, $\Gamma \in R^{2\times 3}$ and $\Sigma \in R^{2\times 2}$ are the unknown parameters of the system.

We can write *the structural form* of the model (1) more compactly as

$$Y\mathrm{B}' = X\Gamma' + E \tag{2}$$

where Y, X and E are the matrices with rows y_i', x_i' and e_i', respectively. The vectors x_i' s are fixed and uncorrelated with the e_i' s.

Hence,

$$\begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix} = \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix} \begin{bmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{22} \\ 0 & \gamma_{23} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{3}$$

Model specification includes restrictions on Γ and B such that some coefficients must be 0 or 1 without which they would not be identifiable.

The *reduced form* of the model $By_i = \Gamma x_i + e_i$ which is equivalent to $y_i = B^{-1}\Gamma x_i + B^{-1} e_i$

is given by $$y_i = \Pi x_i + u_i \tag{4}$$

where $\Pi = \mathrm{B}^{-1}\Gamma$ and $u_i = \mathrm{B}^{-1} e_i$

The matrix Π can be consistently estimated from (4) by applying the least squares estimator (LSE) to each coordinate, but one cannot in general get Γ and B from it, except in certain circumstances ("exact identifiability").

For our study, we chose the model:

(5)

where y_1 and y_2 are endogenous, x_1, x_2 and x_3, standard normally distributed, exogenous and u_1, u_2 autoregressive of order one, with varying parameters 0.3, 0.6 and 0.9. The initial parameters were arbitrarily chosen as a = 0.5, b = 0.8, c = d = e = 1. The first and second equations are just identified and over identified respectively.

2.1 Estimation Methods

The parameters of a structural equations of a system of simultaneous equations can be estimated either by single equation methods where one equation of the system is solved at a time, or by complete systems techniques where the solution of all equations in the system is done simultaneously and the estimation of the parameters of all the coefficients of the system are solved for at the same time.

Simultaneous equations models are an important tool in Econometrics. They are an extension of the multivariate linear model (MLM). While their correctness in specific situations may be open to criticisms, there is no doubt that they constitute an interesting field of research for statisticians. In particular, research on the effects of outliers on estimation procedures, and on methods robust with respect to outliers, seems to be scanty (Maronna and Yohai, 1994).

The choice of estimation technique is somewhat dependent on many factors ranging from the identifiability status of the equations of the model, the available information concerning the other equations of the system, various statistical properties of the parameters estimates, the computational complexity of the technique, the magnitude of the perturbations or outliers present in the data and several other factors Koutsoyiannis (2001).

Another factor that is useful in the ranking of the various techniques, namely the general rule that estimators which are obtained from methods using more information are more efficient. The single equation estimation methods lead to estimates that are consistent but generally not asymptotically efficient. The reason for the lack of asymptotic efficiency is the disregard of the correlation of the disturbances across equations. Another explanation for the lack of asymptotic efficiency is that single equation estimators do not take into account prior restrictions on other equations in the model (Kmenta, 1971). In general, it is intuitively clear that the more information we use in estimating a structural parameter, the more efficient the estimate will be, that is the closer they are to the true parameter (Koutsoyiannis 2001).

All simultaneous equation estimation methods have some desirable asymptotic properties. These properties become effective in large samples, but since samples are mostly small in practice (Kmenta, 1971 and Johnston, 1972), we would be more interested in knowing the small sample properties of these estimators.

Theoretical ranking of the various econometric techniques on the basis of the asymptotic properties is important when the sample size is sufficiently large. However, as mentioned earlier, researchers seldom get large samples hence they usually work with small samples, the asymptotic properties of the estimates are of little assistance in their choice of technique.

Traditionally, the ranking has been based on some ‘small-sample properties’ which are considered as ‘desirable’ or ‘optimal’ for the estimate to possess (Adepoju and Olaomi, 2009). These properties are unbiasedness, minimum variance, minimum mean square error and the proportion of wrong inferences about the significance of the parameters by using a particular econometric method (Koutsoyiannis 2001). The problem of choice of estimation technique is by no means a simple task. This problem has been discussed to a great extent in econometric literature, yet no conclusive evidence as to the ranking of the various

econometric techniques has been achieved (Adepoju and Olaomi; 2009) especially when the available data is plagued with the problem of influential observations or outliers.
We give a brief explanation of each of the methods used in the estimation process of our model.

2.1.1 Ordinary Least Squares

This technique minimizes the sum-of-squared residuals for each equation, accounting for any cross-equation restrictions on the parameters of the system. If there are no such restrictions, this method is identical to estimating each equation using single-equation ordinary least squares. It is not consistent, for the regressors are not uncorrelated with the disturbances.

2.1.2 Two-Stage Least Squares

The two-stage least squares (2SLS) is an appropriate technique when some of the right-hand side variables are correlated with the error terms, and there is neither heteroskedasticity, nor contemporaneous correlation in the residuals. The method of *two-stage least squares (2SLS)* avoids this pitfall of the OLS by first regressing the *y's* on the *x's* (first stage) and then estimating the parameters by applying OLS (with the restrictions) to (3), but with the *y's* on the right-hand side replaced by the fitted values $\widehat{Y} = X'X^{-1} X'Y$ (second stage). This method is consistent. However, it is in general not asymptotically efficient, for the estimation for equation *j* does not take into account the information contained in the other equations.

2.1.3 Weighted Two-Stage Least Squares

The weighted two-stage least squares (WTSLS) estimator is an appropriate technique when some of the right-hand side variables are correlated with the error terms, and there is heteroskedasticity, but no contemporaneous correlation in the residuals.
TSLS is first applies to the unweighted system. The results from this estimation are used to form the equation weights, based upon the estimated equation variances. If there are no cross-equation restrictions, these first-stage results will be identical to unweighted single-equation 2SLS.

2.1.4 Three-Stage Least Squares

Three-stage least squares (3SLS) is the two-stage least squares version of the Seemingly Unrelated Regression method. It is an appropriate technique when right-hand side variables are correlated with the error terms, and there is both heteroskedasticity, and contemporaneous correlation in the residuals.
2SLS is applies to the unweighted system, enforcing any cross-equation parameter restrictions. These estimates are used to form an estimate of the full cross-equation covariance matrix which, in turn, is used to transform the equations to eliminate the cross-equation correlation. 2SLS is applied to the transformed model.

2.1.5 Generalized Method of Moments (GMM)

The GMM estimator belongs to a class of estimators known as M-estimators that are defined by minimizing some criterion function. GMM is a robust estimator in that it does not require information of the exact distribution of the disturbances.

GMM estimation is based upon the assumption that the disturbances in the equations are uncorrelated with a set of instrumental variables. The GMM estimator selects parameter estimates so that the correlations between the instruments and disturbances are as close to zero as possible, as defined by a criterion function. By choosing the weighting matrix in the criterion function appropriately, GMM can be made robust to heteroskedasticity and/or autocorrelation of unknown form.
Many standard estimators can be set up as special cases of GMM. For example, the ordinary least squares estimator can be viewed as a GMM estimator, based upon the conditions that each of the right-hand side variables is uncorrelated with the residual.

2.2 Experimental Framework

The small sample properties of the various econometric techniques have been studied from simulated data in what are known as *Monte Carlo Studies,* and not with direct application of the techniques to actual observations. We use Monte-Carlo approach for the investigation due to the fact that when the covariance between the independent variable and the autocorrelated error terms is non-zero, the problem is near intractable by analytical procedure. Also the properties of FGLS estimators vary depending on the form of the variance – covariance matrix, and often the quality of this variance – covariance matrix cannot be neatly summarized. (See Fair [9, 10]).
This study is thus conducted using a Monte Carlo Experiments using a two-equation model of a just identified and an over identified equations. The degree of autocorrelation affects the efficiency of the estimators (Kmenta, 1971 and Johnston, 1972). Consequently, we investigate the sensitivity of the estimators to the degree of autocorrelation by varying rho (ρ) from 0.3, to 0.6 and 0.9. We also found out the effect of the outliers using three scenarios of no outlier, single and double outliers injected into the endogenous variable. correlation of the independent variable and the error terms at significant level 1%, 2% and 5% on the estimators. The effect of sample size was also investigated by varying the sample size (N) from 15, 20 and 25 each replicated 10,000 times. Evaluation of the estimators was done using the Bias, Variance and the RMSE criteria.

Using model (5), a value U_o (for specified sample size) was generated by drawing a random value ε_o from N(0,1) and dividing by $\sqrt{(1-\rho^2)}$. Successive values of ε_t drawn from N(0,1) were used to calculate an autoregressive U_t. Each X_t was generated as N(0,1) variates and fixed while the initial values for y_1 and y_2 were arbitrarily chosen to be N(0,1) random values. The initial parameters were arbitrarily chosen as a = 0.5, b = 0.8, c = d = e = 1. This procedure is repeated for all ρ, n and pollution levels. $Y_{ts.}$ are thus computed using the model (5). A total of 10,000 replications were performed for each sample size. The data generations and estimations were done via the E-Views package.

3.0 Simulation Results

The results are presented in three tables. Table 3.1 to 3.3 shows the table of average biases, sum of variances and sum of RMSE of estimates for n = 15, n = 20 and n = 60 respectively. We present the ranking of the various estimation techniques when the data are contaminated with various magnitudes of outliers.

The preliminary result, using the univariate GLM, revealed that, using the Bias, variance and the RMSE criteria on both equations, the estimates differ significantly from each other. The estimates from 3SLS and GMM are not significantly different but each significantly different from the other methods; estimates from OLS, 2SLS and W2SLS are not significantly different from each other using both Variance and RMSE criteria while OLS estimates are significantly different from those of the other methods using the Bias criterion. On the sample size n, in the Bias criterion, the estimates are significantly different from each other by sample size while estimates at n=15 and n = 25 are significantly different from each other but each not significantly different with those at n= 20 in the Variance and RMSE criteria. The estimates also differ by pollution levels (Pol_Lev) as there is significant difference in the level of pollution. We observe a surprise result in the estimates by autocorrelation level, at all criteria types, Bias, Variance and RMSE, the estimates are not significantly different.

Looking at Equation 2, the variance of the estimates by all classifications is not significantly different, while in the RMSE, the 3SLS estimates are significantly different from others but GMM. The estimates Biases show 3SLS significantly different from others while GMM and OLS do not differ significantly but GMM differ significantly from W2SLS and 2SLS. Pollution level significantly different from each other and estimates at n=15 differ significantly from those at n=20 and n=25, both the later insignificantly different.

Using the minimum criteria of Bias, Variance and RMSE, It could be seen that identical results were obtained for the 2SLS and W2SLS methods, hence in summarizing the results, both methods were combined using the abbreviation 2WSLS (that is, 2SLS and W2SLS).

Comparing the system methods with the single equation methods, we observed that the latter clearly performed better than the former. Generally, the estimates obtained for equation one (just identified equation) are better than those of equation two, the over identified equation.

Surprisingly, the ranking of the various techniques on the basis of their small sample properties does not reveal any distinguishable feature according to whether there is outlier(s) in the data or not and at the different level of correlation.

On the criterion of the BIAS the best method is OLS in the first equation, followed by 3SLS in most cases especially where the pollution level is zero for all the three autocorrelation levels considered. The GMM and 2WSLS struggle for the third and last positions. However, in the over identified case, 3SLS is leading closely followed by GMM in most cases (when rho = 0.9 for all sample sizes considered) and OLS in few other cases (especially at rho = 0.3 and 0.6 and for N = 20 and 25 with single/double pollution levels).

OLS also ranks highest on the criterion of variance for both equations, but this has little merit since the variance is measured around a “wrong” biased mean. In both equations, OLS is conspicuously followed by 2WSLS with GMM and 3SLS interchangeably assuming the third and last positions.

Using RMSE as the basis for ranking the performance of these methods, OLS is clearly ranks first followed by 2WSLS in most cases.

Conclusion

We have investigated the effects of outliers on a two-equation simultaneous model, where one equation is just identified and the other is over identified. Surprisingly, the ranking of the various techniques on the basis of their small sample properties does not reveal any distinguishable feature according to whether there is outlier(s) in the data or not and at the different level of correlation. This corroborates (Mishra, 2008) that these effects are so intricately pervasive that it is very difficult to assess the influence of outliers on the estimated structural parameters. It is expected that we would be able to identify or suggest the best method to use when we have the scenario depicted above. From the experiment, we recommend system equation method for estimation and if possible, models should be made to be just identified.

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Table 3.1: PERFORMANCES OF ESTIMATORS BASED ON BIAS CRITERION

		n -->	15			20			25		
Eqn	**Method**	**Rho\Pol_Lev**	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	OLS	0.3	-2.388	-2.357	-2.627	-2.408	-2.388	-2.615	-2.297	-2.262	-2.525
		0.6	-2.381	-2.330	-2.612	-2.397	-2.356	-2.594	-2.300	-2.242	-2.500
		0.9	-2.362	-2.247	-2.571	-2.360	-2.251	-2.520	-2.311	-2.182	-2.444
	2SLS	0.3	-2.265	-2.045	-2.618	-2.286	-2.105	-2.684	-2.172	-1.949	-2.418
		0.6	-2.274	-2.023	-2.593	-2.288	-2.085	-2.651	-2.203	-1.961	-2.403
		0.9	-2.285	-1.963	-2.507	-2.281	-1.997	-2.545	-2.290	-2.038	-2.431
	3SLS	0.3	-2.283	-2.005	-2.352	-2.301	-2.142	-2.554	-2.192	-1.990	-2.219
		0.6	-2.307	-2.027	-2.408	-2.322	-2.158	-2.520	-2.243	-2.057	-2.312
		0.9	-2.330	-2.111	-2.533	-2.364	-2.220	-2.487	-2.390	-2.354	-2.609
	GMM	0.3	-2.258	-2.049	-2.433	-2.291	-2.143	-2.539	-2.179	-2.149	-2.358
		0.6	-2.270	-2.036	-2.413	-2.297	-2.130	-2.492	-2.215	-2.132	-2.396
		0.9	-2.282	-1.994	-2.411	-2.303	-2.070	-2.378	-2.324	-2.138	-2.568
	W2SLS	0.3	-2.265	-2.045	-2.618	-2.286	-2.105	-2.684	-2.172	-1.949	-2.418
		0.6	-2.274	-2.023	-2.593	-2.288	-2.085	-2.651	-2.203	-1.961	-2.403
		0.9	-2.285	-1.963	-2.507	-2.281	-1.997	-2.545	-2.290	-2.038	-2.431
2	OLS	0.3	-0.014	-0.967	-1.024	0.006	-0.784	-0.773	-0.001	-0.634	-0.962
		0.6	-0.029	-0.973	-1.029	0.054	-0.715	-0.692	0.001	-0.601	-0.937
		0.9	0.056	-0.834	-0.891	0.494	-0.121	-0.074	0.036	-0.135	-0.435
	2SLS	0.3	-0.021	-0.896	-1.148	-0.011	-0.608	0.180	-0.004	-0.229	-0.712
		0.6	-0.040	-0.910	-1.192	0.036	-0.554	0.297	0.001	-0.186	-0.718
		0.9	-0.016	-0.857	-1.127	0.413	-0.085	1.120	0.034	0.334	0.576
	3SLS	0.3	-0.157	-1.303	-1.836	-0.238	-1.188	-0.220	-0.150	-0.836	-1.351
		0.6	-0.692	-1.326	-1.885	-0.257	-1.152	-1.006	-0.279	-0.768	-1.536
		0.9	-0.473	-1.299	-2.060	-0.292	-0.885	-2.628	-0.859	-1.236	-1.858
	GMM	0.3	0.011	-0.874	-1.406	-0.090	-0.593	-2.205	-0.057	-0.239	-0.719
		0.6	-0.193	-0.939	-1.451	-0.050	-0.588	-0.414	-0.089	-0.146	-0.870
		0.9	-0.223	-0.932	-1.611	0.193	-0.252	-2.361	-0.321	-0.061	-0.981
	W2SLS	0.3	-0.021	-0.896	-1.148	-0.011	-0.608	0.180	-0.004	-0.229	-0.712
		0.6	-0.040	-0.910	-1.192	0.036	-0.554	0.297	0.001	-0.186	-0.718
		0.9	-0.016	-0.857	-1.127	0.413	-0.085	1.120	0.034	0.334	0.576

Table 3.2: PERFORMANCES OF ESTIMATORS BASED ON VARIANCE CRITERION

		n -->	15			20			25		
Eqn	**Method**	**Rho\Pol_Lev**	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	OLS	0.3	0.031	0.087	0.128	0.021	0.056	0.090	0.015	0.028	0.050
		0.6	0.032	0.079	0.119	0.024	0.052	0.084	0.016	0.028	0.050
		0.9	0.053	0.088	0.103	0.046	0.062	0.068	0.027	0.034	0.046
	2SLS	0.3	0.444	0.452	0.527	0.366	0.465	0.757	0.330	0.841	0.547
		0.6	0.341	0.414	0.654	0.321	0.442	0.726	0.274	0.745	0.619
		0.9	0.464	0.577	0.765	0.396	0.645	0.766	0.203	0.287	0.304
	3SLS	0.3	0.531	0.575	1.428	2.152	0.450	1.021	0.632	0.879	1.081
		0.6	0.458	0.550	3.683	0.700	0.382	1.307	0.415	0.918	1.059
		0.9	11.225	1.280	1.984	1.204	2.138	1.297	0.666	0.446	0.510
	GMM	0.3	0.783	1.062	1.392	1.495	0.645	2.270	0.631	0.791	1.067
		0.6	0.681	1.422	1.980	0.605	0.499	1.690	0.398	0.678	0.895
		0.9	3.994	1.121	3.563	0.691	1.120	0.771	0.445	0.451	0.365
	W2SLS	0.3	0.444	0.452	0.527	0.366	0.465	0.757	0.330	0.841	0.547
		0.6	0.341	0.414	0.654	0.321	0.442	0.726	0.274	0.745	0.619
		0.9	0.464	0.577	0.765	0.396	0.645	0.766	0.203	0.287	0.304
2	OLS	0.3	0.201	0.680	0.752	0.143	0.398	0.414	0.116	0.284	0.388
		0.6	0.270	0.771	0.867	0.207	0.467	0.447	0.159	0.323	0.429
		0.9	0.647	1.292	1.419	0.536	0.775	0.724	0.404	0.512	0.655
	2SLS	0.3	0.744	0.661	1.891	0.476	0.384	8.451	0.329	0.396	0.844
		0.6	6.164	0.795	2.172	0.787	0.477	10.790	0.777	0.851	1.910
		0.9	2.962	1.449	9.736	2.668	1.162	19.272	2.430	2.070	12.251
	3SLS	0.3	4.774	0.913	169.147	4.945	0.704	2768.421	0.744	19.334	5.694
		0.6	40182.668	1.207	8.249	5.481	0.866	7583.066	4.154	16.364	14.484
		0.9	21.971	1.808	88.460	38.779	2.149	1317.523	17.383	56.396	752.206
	GMM	0.3	17.698	0.825	16.998	3.359	0.656	8131.088	0.856	3.000	6.258
		0.6	987.136	0.977	7.324	2.645	0.717	5689.112	1.512	28.388	16.494
		0.9	9.426	1.730	193.525	11.074	1.682	1590.237	9.304	75.199	1296.711
	W2SLS	0.3	0.744	0.661	1.891	0.476	0.384	8.451	0.329	0.396	0.844
		0.6	6.164	0.795	2.172	0.787	0.477	10.790	0.777	0.851	1.910
		0.9	2.962	1.449	9.736	2.668	1.162	19.272	2.430	2.070	12.251

Table 3.3: PERFORMANCES OF ESTIMATORS BASED ON RMSE CRITERION

		n -->	15			20			25		
Eqn	**Method**	**Rho\Pol_Lev**	**0**	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	OLS	0.3	2.406	2.398	2.672	2.421	2.414	2.648	2.307	2.278	2.545
		0.6	2.399	2.368	2.655	2.411	2.382	2.625	2.311	2.258	2.521
		0.9	2.394	2.295	2.611	2.387	2.287	2.549	2.328	2.204	2.465
	2SLS	0.3	2.559	2.369	2.810	2.523	2.423	2.935	2.436	2.621	2.672
		0.6	2.499	2.334	2.835	2.497	2.401	2.901	2.408	2.568	2.701
		0.9	2.567	2.406	2.818	2.533	2.500	2.839	2.407	2.275	2.586
	3SLS	0.3	2.614	2.414	2.917	3.249	2.428	2.906	2.622	2.626	2.767
		0.6	2.583	2.409	3.611	2.701	2.404	2.989	2.520	2.654	2.820
		0.9	5.203	2.794	3.258	2.954	3.196	2.979	2.718	2.581	2.824
	GMM	0.3	2.732	2.719	2.962	3.016	2.555	3.278	2.620	2.655	2.849
		0.6	2.679	2.854	3.150	2.647	2.473	3.086	2.493	2.582	2.821
		0.9	3.778	2.701	3.650	2.692	2.775	2.701	2.563	2.426	2.730
	W2SLS	0.3	2.559	2.369	2.810	2.523	2.423	2.935	2.436	2.621	2.672
		0.6	2.499	2.334	2.835	2.497	2.401	2.901	2.408	2.568	2.701
		0.9	2.567	2.406	2.818	2.533	2.500	2.839	2.407	2.275	2.586
2	OLS	0.3	0.730	1.977	2.104	0.619	1.667	1.729	0.551	1.354	1.654
		0.6	0.847	2.044	2.188	0.747	1.733	1.765	0.640	1.367	1.679
		0.9	1.345	2.348	2.507	1.345	2.033	2.072	1.019	1.405	1.817
	2SLS	0.3	1.475	1.835	2.907	1.183	1.512	4.820	0.977	1.301	1.863
		0.6	4.217	1.945	3.077	1.519	1.610	5.285	1.492	1.688	2.543
		0.9	2.937	2.365	5.642	2.835	2.189	7.226	2.650	2.542	5.994
	3SLS	0.3	3.599	2.141	22.468	3.667	1.901	83.784	1.436	7.473	4.366
		0.6	338.595	2.349	5.443	3.949	2.009	131.260	3.316	5.931	6.766
		0.9	7.480	2.690	16.206	10.490	2.714	56.760	7.000	12.600	45.690
	GMM	0.3	6.867	1.917	7.341	3.061	1.661	145.580	1.521	2.977	4.380
		0.6	53.033	2.060	5.082	2.692	1.734	114.023	2.058	7.608	6.974
		0.9	5.150	2.551	23.878	5.574	2.451	64.437	5.201	13.958	61.316
	W2SLS	0.3	1.475	1.835	2.907	1.183	1.512	4.820	0.977	1.301	1.863
		0.6	4.217	1.945	3.077	1.519	1.610	5.285	1.492	1.688	2.543
		0.9	2.937	2.365	5.642	2.835	2.189	7.226	2.650	2.542	5.994