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Linear Superposition of Symmetric IFS-Based Attractors and Fractal Characterization

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ABSTRACT

This study exploited the possibility of new fractal creation in order to increase drastically the stock number of handy fractal images through the combination of limited base symmetric attractors' codes. The randomised play of 'chaos game' with the iterated function systems (IFS) comprising finite set of contractive affine maps coupled with simple coordinate transformation and linear superposition provide a framework for the new fractal image creation. However the fractal characterization that captures fractal image structural complexity and beauty was achieved by the implementation of optimum disk count algorithms. Comparison of the corresponding analytical and estimated fractal dimension of four symmetric base attractors are within the range of 3.2 and 7.1 percent absolute relative error. The correlation coefficient being R^2=0.97. Aesthetically valuable symmetric fractal images were produced across various combinations explored with estimated fractal dimension $1.5329 \le D \le 1.8156$ at transformation square window size of 2. Estimated fractal dimensions and magnitude were found to be independent of window size and number of base attractors' codes combined respectively. The findings of this study have potential applications in textile industries and general fashion design specializations.

Keywords: *Fractal, Superposition, Symmetric IFS, Chaos Game and Optimum Disk*

1. INTRODUCTION

Fractals can be defined as rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced copy of the whole (Oldřich et al 2001).Equally, Guoqiang (2001) explained fractals as tenuous spatial objects whose geometric characterization includes irregularity, scale-independence, and self-similarity. The application of fractals for dynamics characterization is presently attracting more researchers' interest in engineering and host of other disciplines (medicine, agriculture, languages e.t.c).A detail study of a book by Edward (1996) on invitation to dynamical systems strengthens researchers' interest in fractal characterization. A comprehensive literatures review shows that fractals characterization with iterated function system (IFS) is a key aspect of fractal dynamics that have been extensively exploited. According to Fadhil (2011), the information age brings some unique challenges to society. New technology and new applications bring new threats and force us to invent new protection mechanisms. So every few years, computer security needs to reinvent itself. In the paper of this author, a new image encoding system utilizing fractal theories was proposed. This approach exploits the main feature of fractals generated by IFS techniques. Two levels of encryption and decryption methods performed to enhance the security of the system; this is based on the fact that all fractal functions use real number to ensure satisfaction of contraction property. The proposed method gives the possibility to hide maximum amount of data in an image that represent the attractor of the IFS without degrading its quality. Also to make the hidden data robust enough to withstand known cryptographic attacks and image processing techniques which do not change the appearance of image. A general method is for enumerating the distinct self-similar sets that arise as attractors of certain families of iterated function systems (IFS) has been presented by Kenneth and John(2000) using group theory to analyse the symmetries of the attractors. Since the conception of automatic fractal image compression, the research on this topic has grown rapidly (Aura and Felipe, 1999). Fractal Image Coding by Multi-Scale Selection Based on Block Complexity has been successfully implemented. This authors' work is intended to provide a new vision on this automatic process by introducing the idea of multi-scale domain-pool classification based on the complexity of the image to be compressed. A pre-processing analysis of this image identifies the complexity of each image block computing its local fractal dimension. The dynamics of strange attractors in drift wave turbulence has been examined by Lewandowski (2003) .A multigrid particle-in-cell algorithm for a shearless slab drift wave model with kinetic electrons has been presented. The algorithm, which is based on an exact separation of adiabatic and nonadiabatic electron responses, is used to investigate the presence of strange attractors in drift-wave turbulence. Although the simulation model has a large number of degrees of freedom, it is found that the strange attractor is low dimensional and that it is strongly affected by dissipative effects. Multifractal chaotic attractors in a system of delay-differential equations modelling road traffic is the hallmark of Leonid et al (2002) paper. The cars move in a closed circuit and the system's variables are each car's velocity and the distance to the car ahead. For low and high values of traffic density, the system has a stable equilibrium solution, corresponding to the uniform flow. Decreasing the density from high to intermediate values, a sequence of supercritical Hopf bifurcations forming multistable limit cycles, corresponding to flow regimes with periodically moving traffic jams was observed. Using an asymptotic technique, it is found approximately that small limit cycles born at Hopf bifurcations and numerically preform their global continuations with decreasing density. It is deduced from this study that chaotic and nonchaotic attractors coexist for the same parameter values and that chaotic attractors have a broad multifractal spectrum.

There is no doubt that an extensive and laudable research efforts have been made by numerous authors on the subject of fractal characterization. Notwithstanding, available literatures do not provide an in depth study of linear superposition of symmetric IFS- based attractors and fractal characterization. Obviously, this is a lacuna that is calling for researchers' attention. The ultimate aim of this paper is to exploit the possibility of creating a new fractal so as to increase significantly the stock number of handy fractal images through the combination of limited base symmetric attractors' codes.

2. METHODOLOGY

Barnsley (1993) uses iterated function systems (IFS) to provide a framework for the generation of fractals images. According to definitions given by Hsueh-Ting Chu and Chaur-Chin Chen (2003), a transform $f: X \to X$ on a metric space (X,d) is called a contractive mapping if there is a constant $0 \le s \le 1$ such that equation (1) holds while an iterated function systems (IFS) consists of a finite set of contractive mappings $w_i =, for\ i = 1,2,\ldots,n,$ denoted as $W = \{X; w_1, w_2, \ldots, w_n\}$ and captured by equation (2) for R^2 metric space. The present study focuses on simulation, superposition and fractal characterization of fractal images with at least one axis of symmetry and that meets the requirements of equations (1) and (2). The superposition of the fractal images was effected on the square window described by equations (3) and (4) using the linear transformation equations (9) and (10) and relevant variables defined by equations (5) to (8). It is important to note that choices of lower and upper limits of window coordinates variables has no significant effect on the fractal characterization of superimposed fractal images.

$$d(f(x), f(y)) \le s.d(x,y) \forall x, y \in X, \quad (1)$$

$$w\left(\begin{Bmatrix} x \\ y \end{Bmatrix}\right) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{Bmatrix} e \\ f \end{Bmatrix} \quad (2)$$

$$-2 \le x^{window} \le 2 \quad (3)$$

$$-2 \le y^{window} \le 2 \quad (4)$$

$$x_{min} = Min(x_i, i = 1,2,\ldots,n) \quad (5)$$

$$x_{max} = Max(x_i, i = 1,2,\ldots,n) \quad (6)$$

$$y_{min} = Min(y_i, i = 1,2,\ldots,n) \quad (7)$$

$$y_{max} = Max(y_i, i = 1,2,\ldots,n) \quad (8)$$

$$x^{window} = -2 + \frac{4}{(x_{max} - x_{min})}[x - x_{min}] \quad (9)$$

$$y^{window} = -2 + \frac{4}{(y_{max} - y_{min})}[y - y_{min}] \quad (10)$$

2.1 Superposition

The IFS-parameters of four (4) fractal images were identified from literature, ranked and tagged correspondingly with integer number between one (1) and four (4). A total of fifteen distinct superposition cases were studied according to combination equations (11) and (12).

$$^{m}C_r = \frac{m!}{r!(m-r)!} \quad (11)$$

$$^{4}C_1 + {}^{4}C_2 + {}^{4}C_3 + {}^{4}C_4 = 4 + 6 + 4 + 1 = 15 \quad (12)$$

2.2 Fractal Characterization

The fractal disk dimension based on optimum disk counted algorithms of Alabi *et al* (2007) and Salau and Ajide (2012) was used for the fractal characterization of this study. It assumed the existence of power law relationships between the independent variable observation scales and the dependent optimum disk counted. The mathematical equivalent is given by equation (13) for independent variable (X), dependent variable (Y), constant of proportionality (K) and the fractal disk characterizing dimension (D).

$$Y = KX^{D} \tag{13}$$

Table 1: IFS-Parameters of four fractal images culled from literature with name and number tag

Function (w_i)	Koch (1)					
	A	B	C	D	e	f
1	0.3333	0.0000	0.0000	0.3333	0.0000	0.0000
2	0.1667	-0.2887	0.2887	0.1667	0.3333	0.0000
3	0.1667	0.2887	-0.2887	0.1667	0.5000	0.2887
4	0.3333	0.0000	0.0000	0.3333	0.6667	0.0000
Function (w_i)	Xmas Tree (2)					
1	0.3333	0.0000	0.0000	0.3333	0.0000	0.0000
2	0.3333	0.0000	0.0000	0.3333	0.6667	0.0000
3	0.3333	0.0000	0.0000	0.3333	0.0000	0.6667
4	0.3333	0.0000	0.0000	0.3333	0.6667	0.6667
5	0.3333	0.0000	0.0000	0.3333	0.3333	0.3333
Function (w_i)	Sierpinski Triangle (3)					
1	0.5000	0.0000	0.0000	0.5000	0.0000	0.0000
2	0.5000	0.0000	0.0000	0.5000	0.5000	0.0000
3	0.5000	0.0000	0.0000	0.5000	0.2500	0.5000
Function (w_i)	Carpet (4)					
1	0.3333	0.0000	0.0000	0.3333	0.0000	0.0000
2	0.3333	0.0000	0.0000	0.3333	0.3333	0.0000
3	0.3333	0.0000	0.0000	0.3333	0.6667	0.0000
4	0.3333	0.0000	0.0000	0.3333	0.0000	0.3333
5	0.3333	0.0000	0.0000	0.3333	0.6667	0.3333
6	0.3333	0.0000	0.0000	0.3333	0.0000	0.6667
7	0.3333	0.0000	0.0000	0.3333	0.3333	0.6667
8	0.3333	0.0000	0.0000	0.3333	0.6667	0.6667

2.3 Simulation Parameters

The simulation was driven with arbitrarily picked random number generating seed value of 9876 starting from arbitrary coordinate's point (1, 0.5) and iterated continuously over 500 and 2000 transient and steady solution points respectively. The disk count was performed in space of ten arbitrary start coordinates points and over each of ten scales of observation systematically tied to the attractor characteristic length. The characteristic length is the distance between farthest coordinate pair on the attractor. Thereafter the optimum disk counted (minimum) determine from the range of ten possibilities per observation scale.

3. RESULTS AND DISCUSSIONS

Figure 1 compare very well with Paul. S. Addison (1997) and serve good for the validation of fractal image computation and characterization computer programme developed for this study.

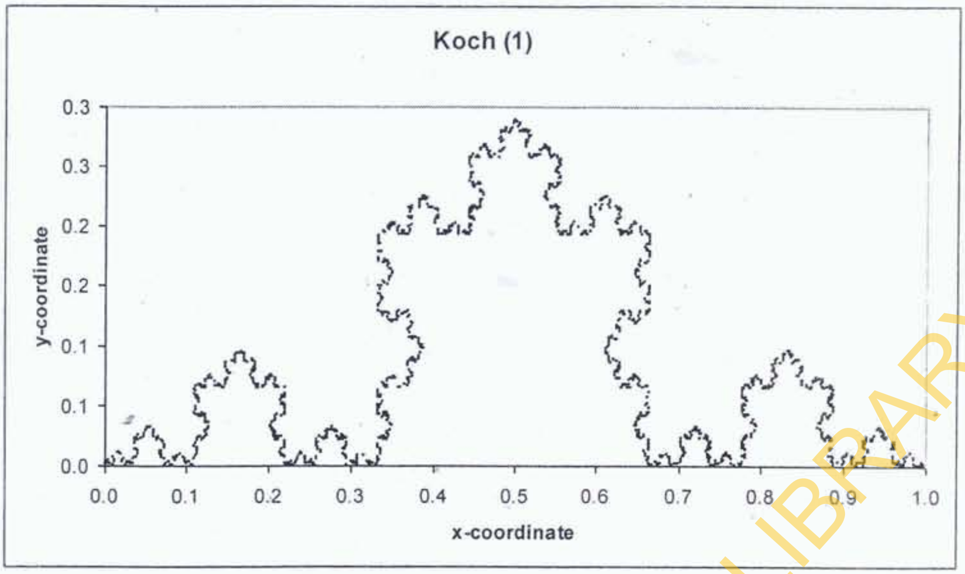

Figure 1: Steady 2000-coordinates IFS-generated fractal image

Table 2: Comparison of analytical and estimated fractal dimension of fractal images

Name/tag number of fractal image	Fractal Dimensions		Percentage absolute relative error
	Analytical	Present Algorithms	
Koch (1)	1.2619	1.3027	3.2
Xmas tree (2)	1.465	1.3993	4.5
Sirpinski triangle (3)	1.585	1.4719	7.1
Carpet (4)	1.8928	1.7583	7.1

Referring to table 2 the present algorithms afforded low range of percentage absolute relative error of 3.2 to 7.1. The trend of fractal dimension using present algorithms compare very well with analytical dimension with coefficient of correlation being R^2=0.97. Thus, the fractal dimension estimate using the present algorithms is a very good alternative method for case situations where analytical dimension is not available or difficult to evaluate.

Table 3: Variation of fractal dimension of new generated fractal with different combination superposed

Case Number	Fractal Dimension of New generated fractal image		Number tags (1-4) of superposed fractal image			
	Present algorithms	Average over individual dimension of superposed images				
1	1.3027	1.3027	1	-	-	-
2	1.3993	1.3993	2	-	-	-
3	1.4719	1.4719	3	-	-	-
4	1.7583	1.7583	4	-	-	-
5	1.6464	1.3510	1	2	-	-
6	1.5329	1.3873	1	3	-	-
7	1.7771	1.5305	1	4	-	-
8	1.5522	1.4356	2	3	-	-
9	1.6807	1.5788	2	4	-	-
10	1.6223	1.6151	3	4	-	-
11	1.6822	1.3913	1	2	3	-
12	1.6763	1.4868	1	2	4	-
13	1.6834	1.5432	2	3	4	-
14	1.6695	1.5110	1	3	4	-
15	1.8156	1.4830	1	2	3	4

Referring to table 3 eleven new additional fractal images were generated with their distinct fractal dimension from only four base fractal images or attracors. Thus, superposition by different fractal images combination can multiply drastically the total number of handy fractal images for aesthetic design purposes without the rigour of searching for new IFS-rules.

The variation of the two estimated fractal dimensions with increasing number of fractal images combination is provided in figure 2. The variations maintain general trend with average fractal dimension consistently lower than estimated dimension by present algorithms. However the deviation between the dimensions generally increases with increasing number of fractal images superposed. The fractal dimension of fractal images that resulted from case number seven (Koch and Carpet) and case number fifteen (Koch, Xmas tree, Triangle, and Carpet) compare very well in magnitude. Thus, the magnitude of fractal dimension of fractal image that result from superposition action is independent of number of attractors superposed. For case ten (Triangle and Carpet), the estimated fractal dimension (1.6223) by the present algorithms is nearly the same with average over superposed attractors. This translates to occurrence of one out of eleven probabilities.

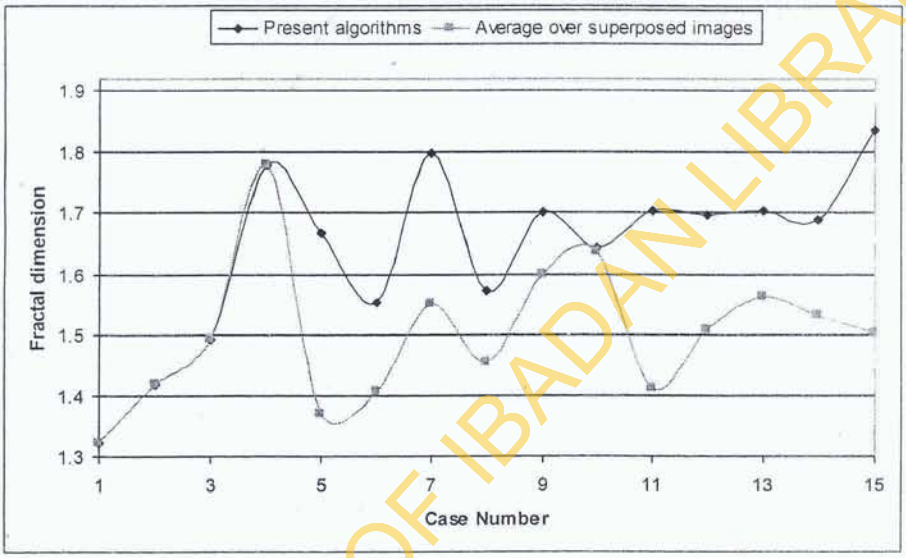

Figure 2: Variation of fractal dimensions with increasing number of superposed fractal images (attractors) combination

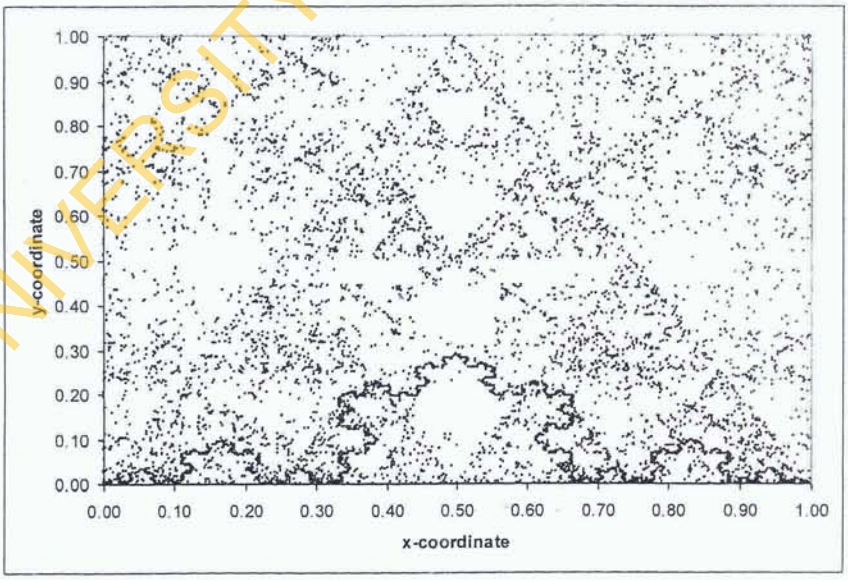

Figure 3: Fractal image result for untransformed superposed Koch, Xmas tree, Sierpinski triangle and Carpet

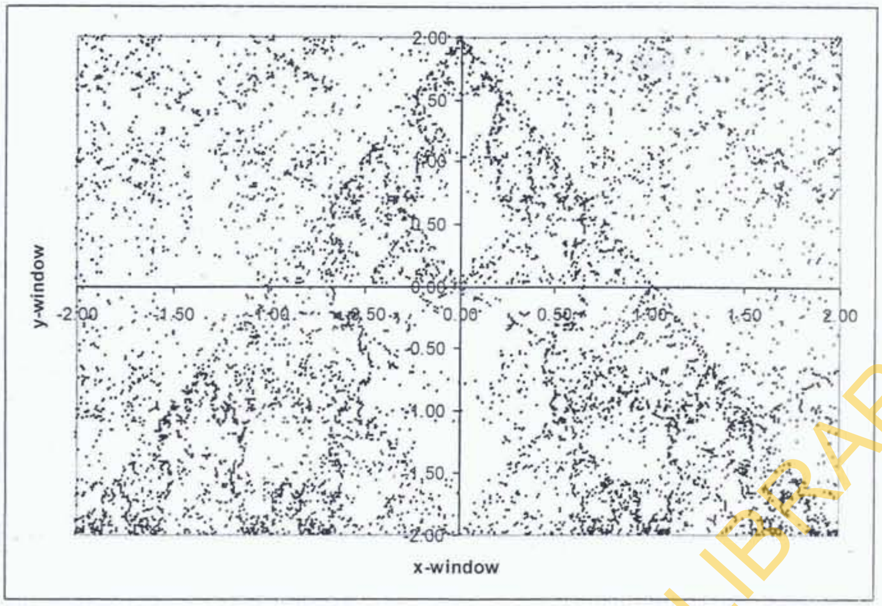

Figure 4: Fractal image result of transformed and superposed Koch, Xmas tree, Sierpinski triangle and Carpet viewed through a square window ($-2 \le x^{window} \le 2$ by $-2 \le y^{window} \le 2$).

Figure 3 and 4 refers. The fractal structures in figure 3 are different and distinct compare to structures found in figure 4, an impact of the transformation factor. Despite the fact that four base fractal images were superposed the wider distribution of grey regions and the outline of the Koch and Sierpinski triangle remain discernable in figures 3 and 4 respectively. In figures 3 and 4 the coordinate range is 0.0-1.0 and -2.0-2.0 respectively. In figure 4 the new fractal image posses' one axis of symmetry and impresses its unique aesthetic value with very rich fractal structures.

Figure 5 gives the variation of the estimated fractal dimension with increasing square window size. The square window size zero corresponds to the untransformed and superposed case situations, similar to case situation reported in figure 3. The estimated fractal dimension is independent of square window size for all cases reported in figure 5. However change in estimated fractal dimension is noticeable from zero to higher value square window sizes for all six arbitrarily selected cases with direction being from lower to higher or vice-versa.

Figure 5: Variation of the estimated fractal dimension with increasing square window size for six (6) arbitrarily selected cases.

4. CONCLUSIONS

This study has shown the possibility of creation of larger number of new, structurally and aesthetically rich fractal images using superposition of transformed coordinate variables of finite symmetric base attractors. The new fractal image has at least one axis of symmetry. Its tolerable estimated fractal dimension and magnitude is independent of square window size and number of base attractors respectively. This study re-affirms that visually pleasant fractal images has associated higher fractal dimension. The findings of this study have potential applications in textile industries and general fashion design specializations.

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